WINNING BELIEFS IN MATHEMATICAL PROBLEM SOLVING

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Abstract: *The aim of this study is to define the relationship between the ability in solving mathematical problems and the beliefs about mathematical problem solving. We compare the beliefs of children with or without difficulties in solving mathematical problems, by using a questionnaire. The results show that good solvers and poor solvers have significantly different concepts of a mathematical problem: some beliefs appear to be winning in that they are able to activate the correct utilization of knowledge.*
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1. Introduction

Several studies in the field of the teaching of mathematics are concerned with the emotional-motivational component of learning (Schoenfeld 1983; Lester 1987; Mc Leod 1992). It has been shown that the failure in solving problems is not only due to the lack of knowledge, but also to the incorrect use of knowledge which is often inhibited by both general and specific beliefs about mathematics.

Beliefs consist of the subjective knowledge that an individual develops in attempt to interpret the surrounding environment (Lester 1987): they influence how the subject learns since they represent the context in which the subject selects and uses cognitive abilities (Schoenfeld 1983; Masi, Poli, & Calcagno 1994).

Some common beliefs of general type associated with Mathematics are (Schoenfeld 1985):

- Mathematics problems are always solved in less than 10 minutes, if they are solved at all.
- Only geniuses are capable of discovering or creating mathematics.

Specific beliefs about mathematics are:
- Since 31>5, then 0.31>0.5.
- The number -a is always a negative number.

Besides the beliefs associated with Mathematics, the beliefs about self are important as well.

Borkowsky and Muthukrishna (1992) have suggested a model that relates behavioral patterns of children facing school tasks, metacognition abilities, and self. In their model, motivation, system of the self, development of correct learning strategies, and self-regulation processes are closely linked. They defined a child with the following features as a good information processor: high self-esteem, internal locus of control (Weiner 1974), causal attribution of success and failure to effort, incremental theory of intelligence (Dweck & Leggett 1988), and feelings of self-efficacy.

Our interest was focused on the emotional and motivational components of this model, in particular the implicit and explicit beliefs about self - self-esteem, role of effort, attributional theories - and learning (both the object of learning and school environment).

More specifically, the aim of our study was to define the relationship between solving mathematical problems, and the beliefs about self and mathematics.

To realize this we have elaborated a questionnaire (the Beliefs about Mathematical Problems Questionnaire) with open and multiple choice questions: in the latter case the proposed options were constructed on the basis of a previous research (Zan 1991 and 1992), that aimed at identifying, through open questions, the conceptual model of real life problems and of school problems possessed by primary school pupils. In the course of this research one of the following three questions was proposed to 750 primary school pupils: "What is a problem for you?", "Give an example of a problem.", and "What comes to your mind when you hear the word *problem*?"

Then we compared the answers to BMPQ given by children with or without difficulties in solving mathematical problems. The findings presented in this paper will concern the beliefs about mathematics, in particular the concept of a mathematical problem.

2. Method

101 children aged from 8.5 to 10.9 participated in this study. The children were in three, four and five grades in the primary schools in Pisa and Livorno, Italy.

The Mathematical Problem Solving Task, characterized by four standard mathematical problems, was presented collectively in order to select two groups of children. Children were classified in two groups: a group of *good solvers* composed of 30 children who solved all the mathematical problems, and a group of *poor solvers* of 21 children who solved one or none of the mathematical problems. Therefore, the final sample was of 51 children.

We administered the Beliefs about Mathematical Problems Questionnaire to investigate the concept of a mathematical problem. Questions were read aloud by the examiner and there was no time limit to answer.

3. Results

Owing to lack of space, we discuss in detail only the answers to the most significant questions. In particular, we omit the answers to the open questions and the justifications for the multiple choice questions, as they would require a more complex analysis.

For simplicity, we have inserted after each question the associated table with the relative data and comments. In order to decide whether the difference between the two groups (good solvers / poor solvers) is significant, we have collected together the answers to some of the questions, naturally on the basis of theoretical criteria.

3. In your opinion, why are mathematical problems called *problems*?
   (Choose just an answer)

[A] This is a usual word to call them: they might also be called “exercises”.

[B] Because for the mind there is a difficult situation to solve.

[C] Because for a child who is unable to solve it, it becomes a problem.

[D] Because they describe someone’s problem and we are asked to solve it.

Question 3 aimed at deciding whether the children reduce the concept of mathematical problem to the more general concept of real problem: this is possible in several ways (answers B, C, D). In case C, unlike B and D, the problematic situation is external to the task and centered on the subject. From a different point of view, the children who choose B recognize as problems a broader variety of situations in comparison with those who choose D. These remarks are summarized in the following table:

Question 3	**Reduction Sch.pr. - Real pr.**		
A	No	**Involvement**	
C	Yes	ego-centered	**Generality**
D	Yes	task-centered	low
B	Yes	task-centered	high

In our opinion, the answers A and C correspond to less effective approaches. Because of their small number, they are collected in one group.

Most good solvers choose answer B, whereas most poor solvers choose answer A or C. [chi^2=6.153; p=0.047]

Question 3	**Good solvers**	**Poor solvers**
A or C	6	11
B	17	6
D	7	4

4. What is a mathematical problem?

[A] It is a text with some numbers and a question.

[B] It is a situation that you can solve by using mathematics.

[C] It is an exercise where one has to decide which operations should be done and then do them.

[D] It is an exercise presented during a mathematics lesson at school.

Question 4	Good solvers	Poor solvers
[A]	4	4
[B]	23	5
[C]	2	6
[D]	1	6

Most good solvers choose answer B, whereas most poor solvers choose answer A or C or D.
[chi^2=13.939; $p<0.001$]

5. Does there exist a mathematical problem without numbers?

Question 5	Good solvers	Poor solvers
Yes	21	8
No	9	13

[chi^2= 5.126; p=0.024]

9. Alessandro says: “A problem with many questions is more difficult than a problem with one question.” Do you agree with him?

Question 9	Good solvers	Poor solvers
Yes	6	11
No	24	10

[chi^2= 5.828; p=0,018]

10. Alice says: “A problem with a short text is easier than one with a long text.” Do you agree with her?

Question 10	Good solvers	Poor solvers
Yes	3	13
No	27	8

[chi^2=13.8 with Yates correction; p< 0.001]

28. In a problem is it worse to make a calculation error or to choose the wrong operations?

[A] Calculation error.

[B] Choose the wrong operations.

[C] It’s the same, there is no difference.

Question 28	Good solvers	Poor solvers
[A]	8	15
[B]	20	5
[C]	2	1

[Considering only answers A and B, chi^2=8.303; p=0.004]

34. How do you feel when the teacher says: “Now let’s do a problem.”?

[A] You are excited but happy.

[B] No particular feeling.

[C] You are nervous because you don’t know if you will be able to solve it.

[D] You are quite scared.

Question 34	Good solvers	Poor solvers
[A]	14	2
[B]	11	1
[C]	1	5
[D]	4	13

We have collected together the answers corresponding to positive or neutral emotions (A and B), and those corresponding to negative emotions © and D).
[chi^2=21.649 with Yates correction; p<0.001]

4. Conclusions

We distinguished four categories of children based on the analysis of the responses given to question 4 of the questionnaire:

1. Formalists - they recognized the mathematical problem on the basis only of the formal features of the text [i.e., it is a text with some numbers and a question];
2. Structuralists - the mathematical problem was identified by using mathematical tools [i.e., it is a situation that you can solve by using mathematics];
3. Operatives - the presence of arithmetic operations defined the mathematical problem [i.e., one has to plan the arithmetic operation and do it];
4. Pragmatist - for these, the mathematical problem was characterized by contextual elements [i.e., it is presented during a mathematics lesson at school].

The good solvers significantly belonged to the category of structuralists.

Furthermore:

- The motivation of good solvers was focused on the task; on the contrary, the motivation of the poor solvers was self-centered;
- The poor solvers were more sensitive to syntactic cues - text length, number of questions, magnitude of numbers - than the good solvers in judging the difficulty of a mathematical problem;
- The poor solvers considered an error in calculation worse than one in planning or selecting the correct arithmetic operations;
- The poor solvers were aware of being anxious when facing a mathematical problem.

In conclusion the good solvers and the poor solvers have a significantly different concept of a mathematical problem.

The difference between the answers of the two groups suggests the following definition. We call “winning” the beliefs of the good solvers, because these beliefs appear to be able to evoke the correct utilization of knowledge.

Metacognitive research has shown that, through a direct intervention, the beliefs about self can be modified. In this way, the cognitive and metacognitive resources can be activated.

Hence, in view of this research, our results suggest a teaching approach to difficulties in problem solving that makes children’s beliefs explicit and endeavours to modify them.

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