GROUP 5:
MATHEMATICAL THINKING AND LEARNING
AS COGNITIVE PROCESSES

INGE SCHWANK (D)
EMANUILA GELFMAN (RUS)
ELENA NARDI (GR)
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Inge Schwank\textsuperscript{1}, Emanuila Gelfman\textsuperscript{2}, Elena Nardi\textsuperscript{3}

\textsuperscript{1}Institute for Cognitive Mathematics, Department of Mathematics & Computer Science, University of Osnabrueck, D-49069 Osnabrueck
schwank@mathematik.uni-osnabrueck.de

\textsuperscript{2}Department of Algebra & Geometry, Tomsk Pedagogical University, Komsomolskii Prospect, 75, 634041 Tomsk, Russia
gefman@mpi.tomsk.su

\textsuperscript{3}University of Oxford, 15, Norham Gardens, OX2 6PY Oxford, U.K.
elena.nardi@linacre.oxford.ac.uk

The work was prepared in such a way that - after the reviewing process - all accepted papers were distributed to the prospected group members. In the spirit of CERME the group leaders decided that during the sessions the accepted papers would not be orally presented one by one. For the purpose of the stimulation of a goal-orientated, in depth discussion six general themes had been identified and two members of the group were asked to give a general introduction referring to the papers fitting each theme and to current research developments. The themes and the introduction presenters are as follows:

1. \textit{The Nature of Cognitive Structures - Introduction and Overview}
   Emanuila Gelfman & Inge Schwank

2. \textit{Individual Styles of Cognition}
   Sara Hershkovitz & Marina Kholodnaya

3. \textit{Cognition and Emotion/Motivation}
   Elena Nardi & Rosetta Zan

4. \textit{Cognition and Language}
   Pierre Luigi Ferrari & Pearla Nesher

5. \textit{Cognition and New Technologies}
   Tatiana Oleinik & Elmar Cohors-Fresenborg

6. \textit{Mathematical Thinking in Modern Conditions – Situated Cognition}
   Jarmila Novotna & Elena Nardi

http://www.fmd.uni-osnabrueck.de/ebooks/erme/cerme1-proceedings/cerme1-proceedings.html
Fig. 1: The Nature of Cognitive Structures - Personal Cognition

http://www.fmd.uni-osnabruceck.de/ebooks/erme/erme1-proceedings/erme1-proceedings.html
1. The Nature of Cognitive Structures  
- Introduction and Overview

1.1 General framework

At the beginning of the group work Inge Schwank gave an introduction and developed a general framework (Fig. 1) in which personal cognition could be considered under different aspects.

One aspect was the role which different kinds of representation of concepts like visual, verbal and sensor motoric play in the process of concept formation and how these are correlated to mental tools. A second aspect dealt with the field of different aspects of mathematical learning like the matter of mathematical knowledge, the specific mental actions in problem solving and the nature of mathematical thinking. It was worked out that these specific mathematical aspects had to be considered in their correlation to other fields, especially cognition and the use of ordinary language. These different aspects of one person’s cognition are controlled by the person’s beliefs. This is the bridge between cognition and the role of emotion, motivation and the person’s personality. Inge Schwank mentioned that these psychological reflections had to be based on some outcomes of neurophysiology and, even more, on neurobiology. All these aspects discuss cognition under the internal perspective of a person. Of course, in studying mathematical teaching and learning processes this point of view has to be broadened to an external perspective in which different aspects of mathematics, didactical materials or teacher strategies are some examples which influence one person’s cognitive processes. A broader perspective comes into consideration if one studies interactions of persons, either among learners or between teachers and learners.

1.2 Individual Preferences  
- Predicative versus Functional Cognitive Structures

In the centre of discussion there was Inge Schwank’s theory concerning the distinction of predicative versus functional cognitive structures and the hypothesis that most people have a more or less strong preference for one of these two cognitive structures in
which they model, understand and solve given external problems. Inge Schwank pointed out that this distinction had its roots in different aspects of mathematics. As one example she explained how famous mathematicians stress the aspects that mathematical concepts have to be understood through the glasses of process orientation and that concepts very often play the role of tools and not of statements. Relations to some of the closest theoretical concepts were discussed, e.g. declarative - procedural knowledge, procepts, APO-Schema (action→process→object - rule).

1.3 Cognitive Experiences - Information Coding

Emanuila Gelfman treated composition of some forms of students’ mental experience: cognitive, metacognitive and intentional. Students’ cognitive experience is paid special attention. In particular, Emanuila Gelfman gave the examples of how, using specially constructed school-texts for 10-15 year old students, we may introduce different forms of information coding: verbal, visual, sensual-sensory. Then she pointed out the role which different ways of information coding plays in students’ intellectual developments and dealt with the advantages and difficulties which arise from verbal versus formal representation of mathematical concepts and problems.

2. Individual Styles of Cognition

This aspect of cognition was tackled under two complementary points of view. Sara Hershkovitz started her presentation from examples of analysing pupils’ behaviour when they are dealing with specific word problems.

In a more general talk Marina Kholodnaya explained - from a psychological point of view - which different aspects of individual styles had to be considered: information coding styles, information processing styles, solving problem styles, epistemological styles. There was a fruitful discussion concerning the question how the theory of Inge Schwank concerning individual preferences for cognitive structures fitted into the general psychological discussion concerning individual styles of cognition.
3. **Cognition and Emotion / Motivation**

In general Elena Nardi and Rosetta Zan stressed the importance which emotion plays in controlling the process of cognition. They pointed out that failure in solving problems is not only due to a lack of knowledge, but also to the incorrect use of knowledge which is often inhibited by both, general and specific beliefs: about mathematics, about self, about mathematics teaching, about social context.

Elena Nardi exemplified research and literature in the field of cognition and emotion and emphasised the inextricability of emotional and cognitive factors in the formation of conceptual structures. Current literature suggests links between attitudes towards mathematics as a lifetime utility tool and the development of conceptual understanding, between problem solving skills and beliefs about the nature of a mathematical problem, between development of formal mathematical conceptual structures and beliefs about the necessity of mathematical proof etc.

Rosetta Zan refined the discussion of affective factors by introducing the distinction and interplay between beliefs, attitudes and emotions. She revisited the submitted papers by pointing out how the cognitive analysis presented in them was substantially incorporating a consideration of beliefs, attitudes and emotions. She subsequently presented a vista of international works in the area and concluded with potential future research questions in the area.

4. **Cognition and Language**

Pearla Nesher gave an overview concerning research in the solving of word problems under the perspective of language. She pointed out that different wordings may induce different mental models of the given problem in the pupil’s mind. The study of these interferences gives fruitful hints for the explanation of pupils misconceptions and difficulties in mathematics lessons.
Pierre Luigi Ferrari analysed typical mistakes and difficulties of senior students while using mathematics language. Pearla Nesher and Pierre Luigi Ferrari put the question of necessity of special work of students for mastering mathematics language.

5. Cognition and New Technologies

Although there had been a thematic group concerning new technologies the group leaders had decided that in this thematic group concerning cognition there should be one discussion concerning the aspect which role the access to new technologies plays for the development of mathematical cognition and the ways of teaching and learning mathematics. Tatiana Oleinik pointed out how in the practical teaching process specific software like Derive and Cabri géomètre foster students’ cognitive growth. She explained how the use of these software packages can support visual thinking as an important aspect of mathematical thinking and how it can support an open end approach in mathematics teaching.

Elmar Cohors-Fresenborg tackled the problem of the session from a more fundamental point of view. He discussed the interaction between external presentations and the use of mental models under the perspective that the use of computers enables a more visual or process-orientated external representation. He pointed out that according to the distinction between predicative versus functional cognitive structures the support of the dynamic aspect of using computers may support the more functional-orientated students while the traditional way of using static formalisations on paper had supported the predicative ones.

In the following discussion there was a strong debate concerning the question how far the use of computer in the mentioned way - really from a cognitive point of view - has to be distinguished from the traditional way of dealing with mathematical representations on paper. It was pointed out that the specificity of a functional mental model cannot be represented in a static representation on paper, e.g. neither the concept of balancing of weight nor the process of switching and its consequence in a circuit.

http://www.fmd.uni-osnabrueck.de/ebooks/erme/erme1-proceedings/erme1-proceedings.html
6. Mathematical Thinking in Modern Conditions – Situated Cognition

In her introduction Elena Nardi pointed out the increasing international interest in Situated Cognition. Jarmila Novotna then opened up four areas of discussion related to the use of relevant projects in the mathematics classroom: time considerations, a definition of a “project”, projects as social problems and the issue of terminology linked with projects.

Subsequently the group debated on the links between situated cognition and the development of formal mathematical reasoning. Possible risks were highlighted with regard to the excessive embedding of the learner’s mathematical reasoning in the situational structure of a project and alternatives were also brought into consideration.

7. Benefit and Outcome of the Working Group

The discussions in the official group sessions induced in several cases the wish for a more private and more intensive discussion with the target to understand the colleagues’ theoretical positions and to go deeper into the different experimental designs.

The discussion showed that there could be different ways of co-operation. One deals with the question how far behind different wording of theoretical explanations there are common aspects. A further intensive discussion was to make clear where the similarities and distinctions between the different theoretical framework lie.

The second possibility of co-operation deals with the idea that an observed behaviour of mathematical activities can be explained by different scientists using their different theoretical explanations: Two pairs of scientific eyes should provide a better picture of the phenomenon which leads to a deeper understanding.

The third aspect for future co-operation concerns the question how to replicate an experimental design in different countries. This experimental approach is common in science, but should be more often used in mathematics education.
Another aspect of further co-operation could be the possibility of adding a specific situation in an experiment to an additional experimental design from a colleague of a foreign country. One benefit of such extension could be that both partners have a better insight into the cultural variances and invariances and the importance of different curricula or school systems for the analysis and understanding of the mathematical teaching and learning processes.