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Cognitive Structures of Algorithmic Thinking

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1. Introduction

During the recent years we have studied how children at primary and early secondary level form concepts in the field connected with computer programming. Our interest was to describe and understand fundamental cognitive processes which are running in someone while he is concerned with the invention and analysis of algorithms. Our methodology is to observe single pupils when they are dealing with algorithmic problems presented by the researcher. In this paper we will try to combine 3 aspects of recent research in this area: the role of external representation of a concept, the inner cognitive structure and individual different cognitive strategies. We are dealing with these 3 questions under the special aspect of algorithmic concept formation but we are convinced that our observations and hypothesis are fundamental for a broader area of mathematical thinking.

2. Fundamental ideas

One fundamental idea concerning the understanding of concept formation processes in the field of computer programming is the following: The invention of an algorithm is regarded as the problem of organizing elementary actions, which the computer has to execute, rather than the structuring of the given problem (COHORS-FRESENBORG 1982). In the beginning this idea was the basis of a course to introduce fundamental ideas of computer programming to children at early secondary level (COHORS-FRESENBORG/GRIEP/SCHWANK 1979,1982).

The second idea is concerning the cognitive strategies. In some case-studies with 10 years old pupils which were inventing automata networks with the didactical material Dynamic Mazes (COHORS-FRESENBORG 1978) we found out, that some of the pupils

prefer a strategy, which we classified as sequential thinking (SCHWANK 1979). The third idea is concerning the role of external representation of an algorithm. Classroom-observations and case-studies had shown that there exist differences, whether an algorithm was invented by playing with match-sticks for organizing actions with natural numbers, or as a computation-network with the Dynamic Mazes, or immediately as a computer-program. An analysis concerning the role of different external representations can be found in COHORS-FRESENBORG (1986).
In SCHWANK (1979) we find the idea that the specific mathematical structure of the Dynamic Mazes, namely that they are sequentially running, may support a specific way of problem solving behaviour: sequential thinking. On the other hand we find the idea, that the three mentioned forms of representing an algorithm form a hierarchie. This was the basis for the lesson courses in the beginning (COHORS-FRESENBORG/GRIEP/SCHWANK 1979, 1982).
The last idea deals with the difference between inventing and analyzing an algorithm. Even in the beginning of teaching algorithms with the Dynamic Mazes in 1975 we find both types of problems: Inventing a network for a given functioning of an automaton and analyzing a given network by writing down its automaton table (COHORS-FRESENBORG 1978).

3. Experimental research

In several pilot studies there was worked out the following design (COHORS-FRESENBORG 1982,1983): In a situation which may be described as a clinical interview a set of tasks is presented to the pupil by the researcher. The different tasks belong to three categories: in a constructive task the pupil has to invent an algorithm, in an analytic task it has to analyse a given algorithm and in a debugging task it has to analyze a given algorithm which contains an error, and after finding the mistake it has to repare it. If the pupil can not solve the given problem, it receives several hints from the researcher, who has a catalоge of diagnostic points which help him to decide which hint he has to give to the pupil. The set of these hints is the same for all investigated pupils. The number of hints, which has been given by the researcher to a pupil, may be regarded as a measure of its success.

3.1 A first Approach

3.1.1 Pilot Studies

In a first pilot study (COHORS-FRESENBORG 1982) there had been found out that there exist pupils which have very different success in constructing our analyzing algorithms. A deeper analysis of the protocols of those problem solving sessions let to the discovery, that some pupils choose very specific kinds of problem solving behaviour. Some of these pupils started as it could be foreseen: They begin their work on the solution by analyzing the given problem, structuring it and trying to build up a conceptual framework in which they build in their preknowledge about previous problems and their solutions. For this behaviour there was created the terminology "conceptual". Different from this behaviour is the following, so called "sequential" which has been mentioned above: Pupils following this strategy are goal-orientated but they start with a first solution before they have completely structured their ideas; they develope their ideas in a dialog with the material; they analyze partial solutions to find the complete solution by modifying them (COHORS-FRESENBORG/KAUNE).

3.2.1 Main Study

After several pilot studies a systematic and sophisticated investigation of these different aspects of algorithmic thinking mentioned above was done by KAUNE (1985). She took one class grade 7 (16 girls, 7 boys, aged 12-14) in a Gymnasium (in this school there are about the upper 25% of the German pupils). Her aim was to investigate the relation between the abilities of the pupils in constructive and analytic tasks, the role of the prefered form of representation of the algorithmic concepts and the prefered cognitive strategy (sequential versus conceptual).

The important results of her studies are the following: She could proof, that there exist pupils with different abilities in constructing or analyzing an algorithm, the external representation describes a world, in which a pupil is thinking, the preference for the match-sticks or the computationel networks of the Dynamic Mazes is for quite a lot of pupils very stable during the problem solving sessions. There exist pupils which have a specific individual preferençe for one of the two cognitive strategies conceptual (8 pupils) and sequential (8 pupils), but there are 5 pupils which change the strategies. There was the interesting result, that there was no correlation between the sequential cognitive strategy and the preference for working with the sequentially running Dynamic Mazes.

3.2 A second Approach

3.2.1 Special Case-Studies

When I was working with a 14 years old deaf boy I saw, that the action-orientated approach to algorithmic concept formation as it was used by COHORS-FRESENBORG/ STRÜBER (1982) to teach deaf pupils was not successfull with this boy. I could not see, that he was interested and able to organize sequences of actions, but he seemed to me to be very sensitive in reflecting relations between different states and their formal description. I only had success with teaching him after I had decided to resume, that this boy preferes a cognitive structure in which the relations between different mathematical objects and their symbolic descriptions form the basis of his thinking. This hypothesis seemed to me to be very strange because such a thinking in relations is normally expressed in mathematics by the use of predicates and this means by the use of language.

In the second case-study I had the chance to teach a very bright 7 years old boy. Contrary to our previous experiences with primary children he did not like to invent algorithms with Dynamic Mazes. He prefered playing with the match-sticks. When I was analyzing his behaviour I found out, that he always tried to arrange certain relations between f.e. the mathematical objects or between the start situation and the goal. But he did not like and he was not successful when he had to think in terms of functioning of a machine.

3.2.2 Further development of a theoretical framework

The analysis of the above mentioned experimental research led to the following hypothesis (SCHWANK 1985): There exist two different cognitive structures in which the thinking processes are expressed: One structure is built up by predicates (relations) and the other one is built up by functions (operations) . If we say, that a person preferes predicative versus functional thinking we say that such a person is translating the external given problem into his personal internal conceptual representation: One preferes predicates, the other one functions. The prefered internal cognitive structure must be distinguished from the prefered cognitive strategy in the sense of COHORS-FRESENBORG and KAUNE: Conceptual versus sequential. This level of cognitive strategy is working on the cognitive structure. By distinguishing this, we now can explain some findings of KAUNE: Some of their pupils with a conceptual strategy prefered the external representation of the functioning networks. Our explaination is that this external representation was matching their prefered internal cognitive structure, the functional one.

3.2.3 First Experimental Tests

The first experimental proof for our hypothesis that our proposed distinction between the predicative/functional cognitive structure and the conceptual/sequential cognitive strategy describes two indenpendent domains of thinking was given by MARPAUNG (1986). In his case-studies with indonesian boys and girls of early secondary level with an experimental design developed from KAUNE (1985) he has found some examples for our hypothesis (see MARPAUNG 1986, p. 85/86).

Our hypothesis may also explain, why some pupils have such difficulties to repare an error in a given network: They do not think in terms of functioning of a machine. Our own now going on research is concerned with weaker pupils (the lowest 35%) in early secondary level. My first impression is the following: Those pupils are more often able to use simple (static) predicates than to understand the functioning of concepts; they are not able to look "into" the mathematical concepts; they prefer a sequential strategy.

4. Prospect

Although the theoretical framework which we have developed here is concerned with the construction and analyzing of algorithms we are convinced, that it is relevant for explaining other fields of (mathematical) thinking. The event, that we have discovered this, being concerned with the explaination of algorithmic thinking, should not only be explained by random: The mathematical field of algorithms is one in which a cognitive structure built up by functions (actions) is a very natural and therefore fruitfull internal representation of the mathematical conceptual framework. But we remind, that outside the world of algorithms there exists a philosophy of mathematics in which description of actions and operations plays an important role.

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