DIDACTICAL ANALYSIS: A NON EMPIRICAL QUALITATIVE METHOD FOR RESEARCH IN MATHEMATICS EDUCATION

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Abstract: *The methods used for the research in Mathematical Sciences Education are common for Psychology, Pedagogy and other related fields. But, to be honest, many of these approaches lead to too punctual results, not very important and with few possibilities to modify substantially the educational practice. The reason could lies in the inadequacy of such methods to cover the complexity of the field, in which many factors are operating in a non isolated way and interconnected to each other through relationships which need to be identified and analysed previously in a global framework under the mathematical knowledge as a common factor. To carry out this previous task, about which we must decide later the usage of the most suitable methods, we have been using a non empirical integrating research procedure, called Didactical Analysis, built up in the conjunction of meta-analysis and qualitative approaches. This paper exposes the principles, the conceptual framework and the techniques that make up this still being studied methodology.*
Keywords: *methodology, meta-analysis, epistemology of mathematics.*

1. Introduction

The methodology deals, in a general way, with *the way we get knowledge about the world* (Denzin; Lincoln, 1994). In this case, it deals with the world of the phenomena in the educational processes of teaching and learning of Mathematics. The study of these phenomena can be raised by following a general process which Romberg (1992, p. 51) summarizes in ten steps represented in bold line's squares in figure 1. From these ten main activities, the author attaches special importance to the first four ones, while the last six ones (from 5 to 10) have to do with the operating or technical part of the process. But, as long as there are more than twenty recognized procedures for developing this second part, we find that the activities expected to be the most important ones in the process seem to be supported, solely and without any added specifications, by the intuition and researcher's skills, by the background of the problem to be researched (Bishop, 1992, pp. 712-714) and by the previous knowledge coming from the scholarly community. These considerations would be enough to guarantee the quality and relevance of the results if the phenomena were not so complex as they really are, if the main factors of that complexity took part of the research in terms of relationships and, finally, if scholars bore in mind a kind of integration of the different perspectives and traditions beyond the mere interdisciplinary approach. But these conditions either are not carried out satisfactorily or there are doubts that it would be like that, which may lead to the so called "feet of clay" research, that is to say, faultless from the operative or technical point of view (activities 5 to 10) but faulty regarding their initial fundamentals and assumptions (activities 1 to 4).

To try to improve this situation in those researches involving a specific mathematical knowledge, we propose to introduce a mechanism of systematic control of the research process fundamentals just between activities 3 and 4 of Romberg´s scheme. It consists in three activities (3.1, 3.2 and 3.3, figure 1) involving a method that we have recently built and used with acceptable results (González 1995, 1998; Ortiz 1997) looking for an adequate response to the complexity and specificity of Educational phenomena in Mathematics.

The procedure is based on the general principles of *meta-analysis* (Glass & McGaw & Smith 1981), the *multivocal revision* (Ogawa & Malen 1991) and what is known as *conceptual analysis* (Scriven 1988). For its name we have chosen the term "didactical analysis", used in other sense to describe "*... the analysis of the mathematics contents, which is carried out at the service of the organisation of its teaching in the educational systems...*"(Puig, 1997, p. 61). In this case we refer to a systematic, genuine and integrating process which gives an own personality to the research in its first steps.

The modification proposed is justified by the following arguments, which will be extensively developed in the next sections: a) the complexity of phenomena, in which many factors are interacting and for which analysis many perspectives and procedures are needed; b) the partial specificity of the field of study, founded in the involvement of mathematical knowledge; c) the lack of the interdisciplinary approach and the need of an integration of knowledge and approaches in order to achieve a greater effectiveness.

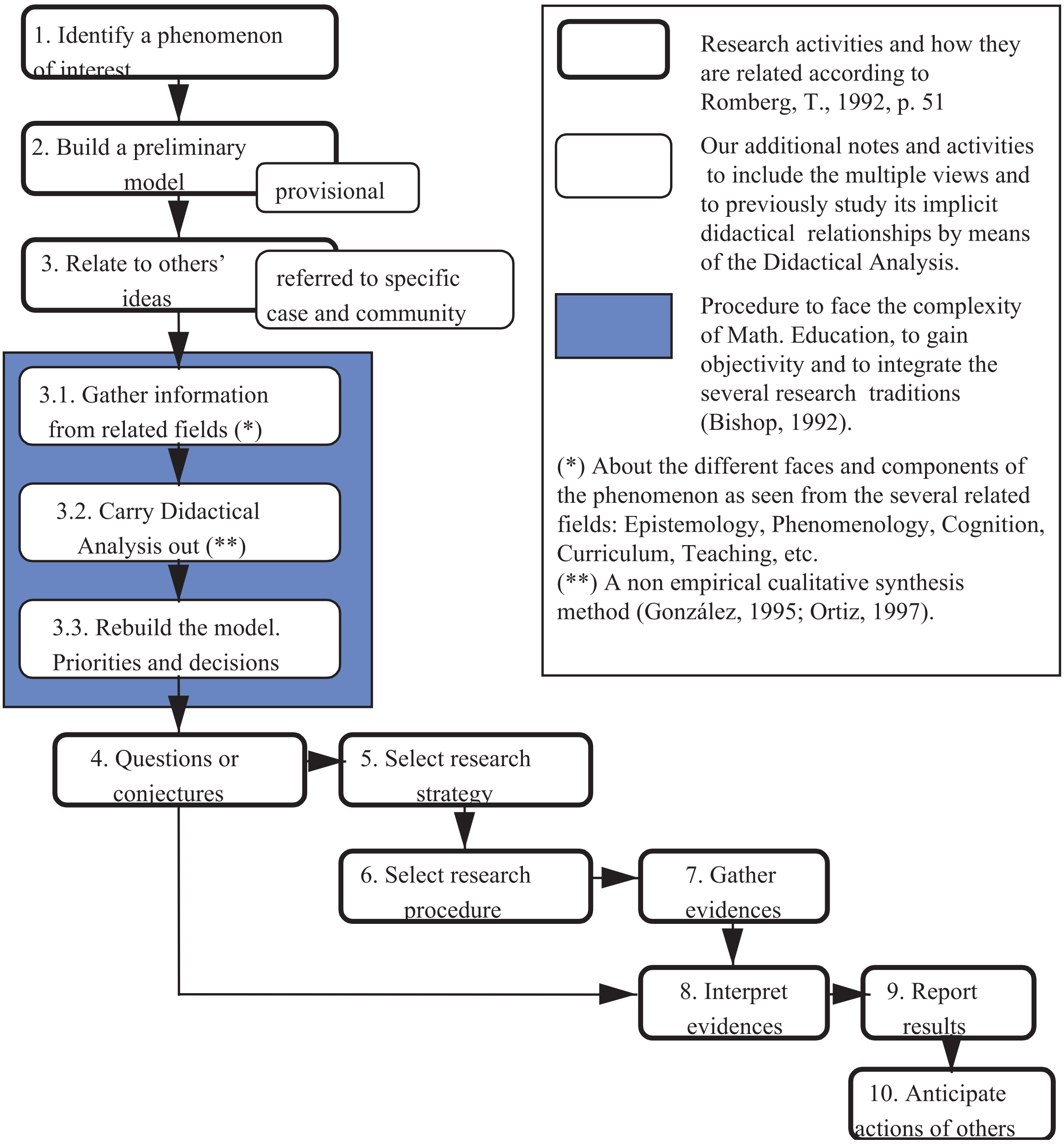

Fig. 1: Research activities and how they would be related according to our experience

2. Mathematics Education: A Field of Relationships vs. Related Fields

In Mathematical Sciences Education, a series of areas, which in the educational practice interact and operate together, can be identified and theoretically separated. Of all these areas we can emphasize, firstly, the one that has to do with *cognitive aspects*, covering, among others, the characteristics and evolution of learns, errors, difficulties and representations and the acquisition of automatism and skills.

On the other hand, we can find a field focussed on teaching, in which specific aspects such as the following: nature, relationships, structure and organisation of the school curriculum (objectives and other organizers (Rico, 1997, pp. 39-59)) witnessing complex factors and conditions (sociocultural, economical, etc.), educational policies, curricular projects and teachers training, can be identified.

Thirdly, we can distinguish a parcel, which is more connected to the real teaching and learning processes, in which some interactions among different factors from the two previous fields are taking part at different levels (design, planning and implementation) (Coriat 1997, pp. 156-157), that is to say: methods for improving learning; resources and materials and curricular adaptations, for instance.

The separation between the above mentioned three fields, which can mainly be observed in the preponderance that researches give to the psychological or pedagogical aspects, seems to be an inadequate approach. The mentioned fields are interconnected to each other and specially related to the Psychology of Mathematics Education (Fischbein, 1990, pp. 6-12) in a first level focussed on the educational finalities and on the general characteristics of mathematical knowledge. Secondly, in special when a specific mathematical topic is taking part, that first level has a tight dependency from other basic elements, as it is the case of Mathematics, its Epistemology and its History or of the Phenomenology of mathematical knowledge; a second level of dependency, which is focussed on both finalities and mathematical contents and based on the following general principles (made up from the considerations of Tymoczko (1986), Davis & Hersh (1988) and Puig (1997)):

a) Mathematical knowledge is perfectible, partial and incomplete, being exposed to failures and having to do with ideas or conceptual objects to which humans accede through the discovery or the non-arbitrary invention or creation. These objects are independent of its symbolisation, having a conventional existence and sharing two different areas: the individual conceptual and the supra individual, cultural or collective as a part of the shared conscience.

b) The phenomena that organize the mathematical concepts are the objects, its properties, the actions on them and the properties of such actions, belonging, all of them, to an unique world in expansion containing the results from human cognition and, in particular, the results from the mathematical activity (Puig, 1997, p. 67).

c) The creation / discovery of mathematical knowledge is conditioned by the common factors to all individuals and cultures that make it possible: the common characteristics of human mind (physiological, for instance), of the environment (physical, social, cultural, for instance) and of the interaction between them.

From the principles and its relationships we draw the following consequences:

1. The involvement of the three factors (mind, environment and interaction) took part in all the interpretations about nature and the way of production of mathematical knowledge, so that the epistemological analysis must take its characteristics into account.
2. The analysis of mathematical knowledge from an educational perspective must include the epistemological, cognitive and phenomenological analysis, which would be related to the sociocultural aspects as well as to teaching and curricular issues as specific and terminal subjects in Mathematical Sciences Education; five major fields which must be involved in the general research framework.
3. The epistemological and phenomenological analysis regarding with educational research must follow a markedly didactical line. The interest must be focussed in obtaining valuable information for teaching and learning, what means thinking about the student, his needs and capacities, about the classroom, its activities and didactical methods. The information gathered through this approach shows the links among the different parts of the two levels mentioned above under a unique reference: *the individual and collective mathematical thought, its evolution, its relationships with other kinds of thought and its education*.

4. This way, the relationships between the Epistemology of Mathematics and the Psychology of Education are in a privileged position, and when focussing all the attention in the processes of creating knowledge it makes sense as a part closely related to mathematical knowledge and to curricular decisions. Likewise, the pedagogical aspect shows a close dependence on the previous factors, to which we should add some other considerations either social, political or cultural that complete a specific and global world view in which multiple relations are needed of a previous integration to carry out the particular studies in the different fields and approaches.

But, such integration must not be finished in a sum of data obtained from different approaches (interdisciplinary conception). On the contrary, it requires a complex elaboration by following a specific research methodology (Didactical analysis) that it must be quite different from the corresponding to the technical part of the process (activities 4 to 10, figure 1). Let´s go to treat next on this argument.

3. Insufficiency of the Interdisciplinary Approach and Specificity

The insufficiency of the interdisciplinary approach regarding Mathematical Sciences Education research, which still has nowadays a wide acceptation by the community although its results can be improved on (Kilpatrick, 1994, pp. 78-79), is based on the relationships among the five major fields and on the nature of the phenomena.

Firstly, the analysis of the relationships may gives new and genuine nuances to the isolated information; this nuances gives cohesion to the study and they provides the integration character we are postulating. Nevertheless, the needs of integration, even importants, not only are justified because of the characteristics of dependencies relations; they can also arise because of the existence of a stagnation of the researches or because of the advanced situation of studies, as it is the case of the concept of function (Harel & Dubinsky, 1992; Romberg, Fennema & Carpenter, 1993) or of the rational numbers (Carpenter, Fennema & Romberg, 1993).

As regards to the nature of the phenomena, we are maintaining the existence of a specific part in the Mathematical Science Education field, to see which it is only needed to analyse the answers to the following questions: Are there important differences between the knowledge of the phenomena in Mathematical Sciences Education when these are analysed from the particular approach of the Didactics of Mathematics and when it is carried out in the context of the general educational phenomena?; Where do such differences lie in?; Is Mathematical Sciences Education a part, in the inclusive sense, of the general field of Education?; If there are any differences, is it correct to use the methods which are usually applied for the non-specific educational research?; Are such methods enough?; Are they priority ones or of preferential application to any other consideration?.

It is not difficult to come to the conclusion that what is characterizing the Mathematical Sciences Education is not the interdisciplinary aspect just like that, but a specific and deeper way of studying the phenomena regarding the teaching and learning processes. This particular point of view can be summarized, in one hand, in the involvement of some specific basic components (the sociocultural as general one and the four remainders as specific and central ones), which have an essential role in the curricular studies (Rico 1997) and among which we emphasize the one that has to do with the *epistemological and phenomenological considerations about the mathematical knowledge in a global framework under didactical purposes*. In the other hand, in the delimitation and analysis of a relationships' network among the four central components (See figure 2) (Gonz·lez et al. (1994); Gonz·lez (1995, 1998)). These two aspects do not intervene in the interdisciplinary approach, which is usually limited to a simple collection of data coming from different approaches.

To sum up: The phenomena regarding the teaching and learning of mathematics show general aspects, which are part of the interest of other disciplines, and specific connotations that introduce differences in the way of approaching the same problems from other fields of knowledge.

Such specificity lies in the important involvement of epistemology and phenomenology of mathematical knowledge in a global framework under educational purposes as well as in its relationships to other fields (pointed out by Vergnaud (1990, p. 22-23)). When taking this into account we can observe an inversion in the assumptions considering Didactics of Mathematics as a specialised branch of General Didactics or of Psychology of Education (Fischbein 1990, pp.6-12).

It is necessary to include a double point of view in the usual research process: a genuine, specific approach, to support and organize the field (activities 3.1, 3.2 and 3.3, figure 1) through a also specific procedure, and a later common interdisciplinary approach to inquiry in the punctual aspects deduced from the previous study (activities 5, 6 and 7, figure 1). Although suitable for particular purposes, the last approach is neither enough nor priority one, but it must be dependent on the results from the previous analysis.

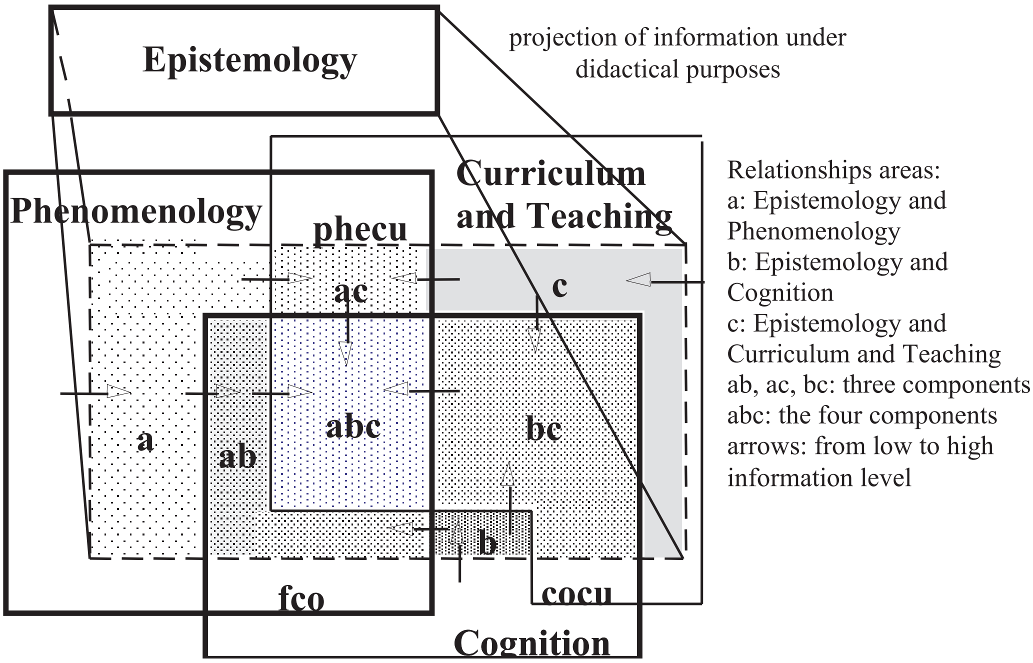

Fig. 2: Relationships' central network

4. Didactical Analysis of a Mathematical Knowledge

In the studies known as *secondary or of synthesis researches* two different methodologies are being used: the *traditional integrating revision* and the quantitative revision, also called *meta-analysis* (Fernández, 1995, p. 165)). Recently, due to the need aroused in many qualitative researches of summarize and integrate a great number of studies, a modality of synthesis, called *multivocal revision* (Ogawa & Malen, 1991) has arisen. It is a procedure of qualitative synthesis "*…aimed at making inquiries on a complex phenomenon of interest in which events can not be manipulated and of which there are many sources of essentially qualitative data, being confident that we can obtain a detailed portrait of the phenomenon under study*". (Fernández 1995, p. 175). The multivocal revision is based on the following criteria, which are similar to the ones suggested for the case studies (p. 176):

1) A clear definition of the research topic by consulting many sources, by keeping evidence's chaines among the records and the inferences drawn and by formally including the informer's reactions to the established conceptual definition.
2) Assess the relative and individual strength of each piece of information by using some of the following criteria: position and certainty of the source (external validity); clearness, detail, consistence and feasibility of the content (internal validity); capacity to corroborate the information through other sources.

Also we are interested in the following criteria regarding the meta-analysis:

3) Revise as many studies as possible; locate them through objective and arguable searches; do not initially exclude studies because of their quality and differentiate and classify each study according to the effect of its results.

The joint consideration of the previous criteria makes up a new approach that we have called *qualitative meta-analysis*. Its finality, as any other meta-analysis, is: "*…the formulation of theories able to explain the phenomena observed in different researches*" (Bisquera 1989, p. 247-252); the difference in this case lies in the use of criteria which are typical from an interpretative approach.

Consequently, we call *didactical analysis* of a specific mathematical topic to the global methodological procedure that integrates and relates, by following a sequential process and according to the criteria of the qualitative meta-analysis, information related to the object of study coming from five basic areas: History and Epistemology, Learning and cognition, Phenomenology, Teaching and curricular studies and Sociocultural aspects. The sequenced process has the following stages:

First stage: Primary revision of the information in every area, following the process in figure 3 and the following steps: a) analysis and classification according to the established criteria; b) gathering the most important data; c) analysis of the relationships among the data; synthesis and conclusions; d) conjectures and research priorities in every area; e) assessment of each one of the area's revision.

Second stage: Analysis of the relationships among central areas according to the diagram in figure 2 and the following process: f) study of the relationships starting from the information of sections c), d) and e) in each area; g) conclusions; h) conjectures and priorities; aspects to research; i) general results and assessment.

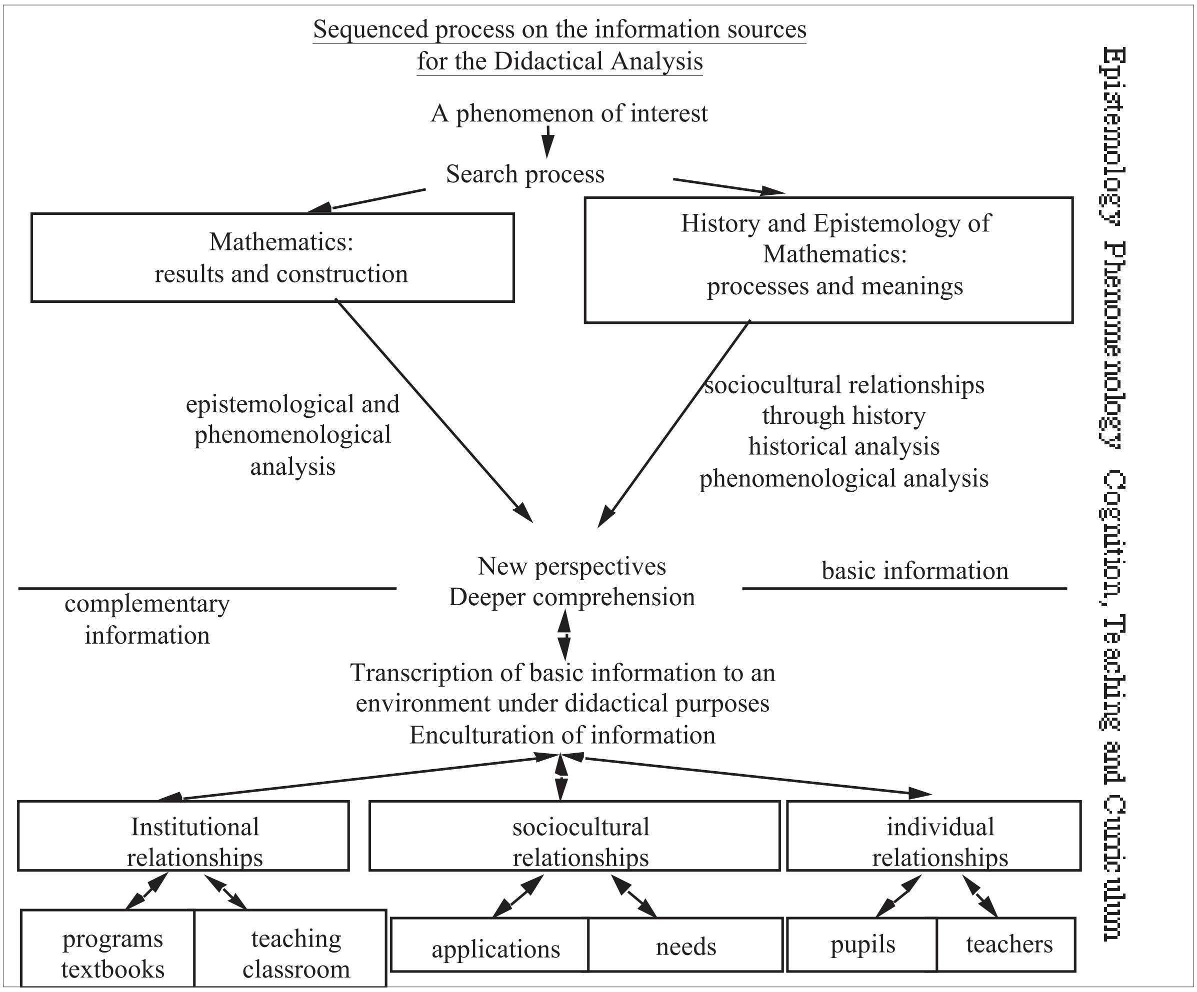

Fig. 3: Integrating information in the first stage of the Didactical Analysis

Figures 2 and 3 describe the basic elements, their position in a sequenced process, the main sources of information and the kind of analysis to be done. Throughout the process data are compared and a global synthesis is carried out. As a consequence, research priorities are established, theories and models are built and empirical and experimental approaches are designed. *The didactical analysis processes, analyses and synthesises information coming from different fields linked to one another by its object of study, giving a synthesis that let the detection of limitations of previous works and properly organizing the future development of the research. The technique used bears in mind the complexity of the field as well as the plurality of approaches and results we can find in the scientific literature.*

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