RESEARCH IN MATHEMATICS EDUCATION: OBSERVATION AND … MATHEMATICS

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Abstract: Mathematics education is a subject whose complexity justifies, in its examination, various break downs into constituent parts. But it is difficult to verify the reintegration of partial studies within the knowledge of global process. This legitimization requires specific concepts and observations, among others clinical observation, both participatory and controlled. The field of research in mathematics education is open to numerous disciplines. Part of research in the didactics of mathematics is mathematical activity aimed at mathematicians, as well as teachers. It can only succeed through interaction with the community of mathematicians.

1. Introduction

I feel very honored that the organizing committee has provided this chance for me to address you all, and I thank its members.

I hope you will forgive me for not providing, as the century comes to an end, a panoramic overview of actions in the domains that concern us.

Since 1960 I have participated in a number of them, and would have evoked with pleasure the projects, the hopes, the actions of some of the principal European architects of research in the field of mathematics education.

These actions are alas too numerous and too complex, their originators still too close to us to allow a short yet satisfactory synthesis, one that would be acceptable to all our diverse points of view. Such an effort calls for more time, resources and talent than I can muster today.
I hope you will also forgive me for not presenting some sort of «Osnabrück program» or an overview of subjects and tasks about to be tackled by the community of young researchers. I am unlikely to do better than Hilbert, and one sees how, even in mathematics, the march of science must submit to contingency and admit of what we might call a historical character, in other words one that escapes our own conception of necessity.

Finally, I will request your indulgence. What I have to say may seem banal to most of you, but this is neither the time nor the place for making scientific revelations. Let me only try and highlight a few comments, a few ideas around which we can come to an understanding. Such understanding might well be of use to the extended fraternity of researchers and participants with which we are dealing.

Thus I will not expound upon the points that seem to you the most urgent and most important.

Given the language and cultural differences, and differences in educative systems, we find constituted within Europe a paradigm that includes almost the whole range of educative propositions in the world at large. All the difficulties and all the obstacles that prevent a common culture and science are present. Problems about research orientation, about joining researchers to institutions, about the organization and diffusion of knowledge produced, all occur at the same level of complexity as those met on a worldwide scale. The members of our association will no doubt prove their usefulness by the way they deal with both the richness and weakness of this situation. For 40 years I have regularly witnessed small teams of researchers in mathematics education move from one institutional connection to another, as dictated by fashion and social conditions, without solving a single problem concerning epistemological foundations and the social legitimization of their work, driven along as ever by immediate necessity.

I’ll limit myself to raising 3 questions.

The first concerns the relationship between a researcher in mathematics education with the subject of research, i.e. the students and their teachers at work.
The second question concerns the role of what in many countries is called didactics\(^3\), in the diffusion and representation of knowledge in mathematics education.

This particularly involves certain forms of observation: clinical “participatory” observation, and controlled experiment.

The third concerns the relationship between the researcher in the didactics of mathematics and mathematics itself, and with mathematicians and their institutions.

When back in the 1960s I first became interested in mathematics teaching, the influence of Jean Piaget made one thing evident: the primary need for greater familiarity with spontaneous cognitive behavior of a mathematical nature, when a child is confronted with a disposition - an environment - unencumbered by didactic intentions. But then the influence of Célestin Freinet, Makarenko, and Vygotsky led me to consider the environment as at once material, social, and cultural. The specific mathematical interactions between the learning subject, and the elements pertaining to this environment, had to be isolated. From these we had to create a model situation, where constraints and evolutions could be studied. This point of departure explains the importance of psychological studies, and the study of cognitive epistomology in the study of mathematics education\(^4\). It also explains their eventual separation from teaching research or mathematics education proper.

2. **Subject of research**

The subject to be studied by researchers brought together here is « mathematics teaching. »

It is a subject at once vast, complex, and very important\(^5\). It would therefore seem imperative to break the subject down into its component parts, insofar as one is not discouraged by the resulting complexity. This breaking down can be done in a number of ways:

- into hypo-systems\(^6\): teachers, students, educational environment, knowledge, teaching and learning, etc.
into infra-systems that can be apprehended within the classic disciplines made available by culture: problems psychological, sociological, linguistic, mathematical, meta-mathematical, epistemological among others, the whole considered in terms of teaching mathematics.

Intensive development in these domains contributes still more to the complexity of approaches, for this multiplies the possibilities of independent studies and indefinitely prolongs the time needed to form researchers in our domain.

This complexity explains the variety of preoccupations of researchers in the realm of mathematics education, their difficulties over delimiting their field and communicating the results of their work, the multiplicity of their institutional ties, their isolation, and their need to come together in society. It also explains both the necessity of ESRME and justifies the diversity of its composition.

But in what measure can knowledge concerning the reductions of the system under study, or established within disciplines that only partially take into account the system, remain valid for the system as a whole? And how can they be integrated and used in didactic actions?

The disappointment caused by the ill-considered importation of knowledge from concepts or theories otherwise firmly established is well known.

For example, is the organization of knowledge adapted to a certain group mathematicians a good « model » for organizing knowledge in the case of a child, or an adult beginner? On the other hand, with what ethical, scientific, experimental right do we « simplify » and transpose cultural knowledge in order to educate, to cause students to be educated according to it?

The again, doesn’t the uncontrolled intrusion and accumulation of dispositions, and the introduction of psychological reasoning into teaching, sometimes have negative effects? To what extent can we use results derived from the observation of a subject, isolated in a one-to-one relationship with a psychologist? Can we predict from them anything about the behavior of the student this subject becomes, once implicated in a process that includes both a teacher and an entire class?
Neither is the simple rationalization of teaching practices and praxeology beyond reproach. Evidence indicates that ill-considered use of evaluations of objectives tends not to reduce, but increase failure.

Established knowledge imported from what we call disciplines « of reference », is necessary and important. Its integration, however, presents unique problems that are part of our field. Perhaps his or her study lies at the very heart of our work, for no one will do it in our stead.

We cannot consider this integration to be transparent, nor a mere question of surface arrangement. Neither can we, a priori and without examination, allows the verification of the validity of our knowledge, relative to mathematics teaching, to rest on the simple juxtaposition of this knowledge of reference. We cannot in particular allow the indispensable empirical link of our subject to depend exclusively on concepts, institutions and methodologies adapted to other ends.

It is recognized - I for one am persuaded - that specific concepts and methods are indispensable, by which I mean ones sufficiently removed from known concepts. Should this indeed be the case, we must provide a forum for debate, a means of accepting or rejecting concepts according to appropriate criteria.

The subject of these concepts would be “ the study of conditions for the diffusion of mathematical knowledge useful to men and their institutions ”, containing within it a more limited project, “ the study of social projects allowing institutions to appropriate, with or without their cooperation, knowledge either already constituted, or on its way to being so. ”

The scientific character of research that has not entirely been “ taken on ” by disciplines of reference, depends on two factors. Certainly it depends on the new concepts being theoretically consistent and compatible with established knowledge in other disciplines. But it depends above all on their valid confrontation with contingency, which implies an interaction between the community with the subject under study, and its meticulous observation.
How is information gathered, how is its validity examined, and how is it then circulated, how does it manifest itself within the community? How is it discussed and transformed, and inversely how does the community act upon education, to satisfy its own need for information?\textsuperscript{10}

It is therefore proper to ask ourselves about the interactions between our academic institutions and the mathematics education “milieu” - in other words the entirety of systems in interaction - which we think resides at the center of phenomena under study\textsuperscript{11}.

The essential lies in the nature of these interactions. Which of them can insure pertinence, validity and sufficiency when the knowledge we work on is put to new uses, whatever its origin?

This point merits particular attention.

3. Forms of observation\textsuperscript{12}

3.1 The subject of observation

The form of mathematics education most currently envisaged is one where a teacher is held responsible for the acquisition, by a group of students - a class - of designated mathematical knowledge. This acquisition occurs within a given lapse of time, and in an environment or milieu, i.e. where learning conditions are determined.

The work of a mathematics class is a good paradigm for all other teaching models\textsuperscript{13}. It therefore adequately represents what we intend to study. Likewise the community can be represented by a single researcher (or by a small group) who takes this class as his subject. The researcher attempts to construct true and transmissible knowledge about conditions that either favor, or inhibit, the teaching process. «New» phenomena are identified. Which interactions between the class and the observer might bring this about, what is the language, the vocabulary and syntax, he can use? The relationship between researcher and subject must be taken into consideration. In any case, what would be of an epistomological (or metascientific) nature in the «hard» sciences, here
becomes, if only provisionally, the subject of the field of study. The researcher is one of the elements of the system, his interactions must be analysed with the same means as those he himself uses to study the interaction between other systems.

3.2 Passive observation

A researcher can remain in strict observation, without personal intervention of any kind on the education process. However, the phenomena he intends to observe do not occur in concentrated form for all teaching processes. Thus observations must be noted in a way that allows the researcher to return to them according to the evolution of his reflections and analyses.

And because his research is habitually the result of a process, or even of a dialectic, it will be in his interest to record as much information as possible. This information will constitute the matter of his study - its « contingency ». His work then becomes that of a paleontologist: to distinguish and extract from this matter what is necessary to separate it from what is, provisionally or residually, of the order of contingency or history. This work already poses interesting theoretical and practical problems. What relation, if any, exists between the observations of a researcher recording from time to time students’ or teachers’ responses to questions specific to his research, and the ongoing process of the class?

Moreover, it is indispensable to create conditions that probably won’t occur naturally, in order to study them. A system delivers more information when it reacts to well chosen stimulation.

Yet to intervene for purely scientific purposes in the educational system, purposes foreign to that system’s mandate, raises important ethical, and consequently methodological and scientific problems.
3.3 The educative action, research - action

Another solution is to integrate one’s research totally into the system. For the researcher this means sharing the different positions of those participating within the system. It also assumes the fact that he intends to transform it, not only in order to understand it, but also to reveal its social function and improve it. The researcher at once studies the entirety of interventions, their actions, and those of other participants and their results.

In the above instance researcher and teacher form a team, each taking on the role of teacher or researcher indifferently. In other instances this fusion will work with the coordinator, the group leader, the inspector, or the minister. This configuration occurs very often. It allows the acquisition of invaluable knowledge. Yet it has frequently led to considerable confusion, and sometimes contributed to very negative interventions when pushed to the extreme.

The value of knowledge acquired in these conditions is more often than not to be accepted with prudence. An assertion is not valid just because the actions associated with it succeed. Nor is it enough for the teacher to have created a few favorable conditions and excelled, etc.

We can see here and there a succession of reforms, founded on ideologies, and on the optimistic appreciation of certain haphazard results obtained by a few « researchers ». But in fact no reform corrects the errors of preceding ones. On the contrary each adds its own pitfalls to the previous ones. Teachers are in no position to integrate and regulate the flood of commands they receive from all sides, but neither can they sustain against them a coherent set of practices. It is very wrong to suppose that at the bottom of everything lie certain “ traditional methods ”, an indestructible adversary and indispensable support for all innovators.

No one wishes to deny teachers the right to seek out improved teaching conditions and better their students’ results. We mean only to adjust the relationship between this effort and the ethics of their profession, its present technical possibilities, and the scientific knowledge of the moment.
The freedom of the intervener, whether researcher or other, to depart from established practice, should be limited by his ability to correct errors eventually resulting from his original action. Such corrections should be effected by established practice.

This freedom implies assuming partial responsibility for results, insofar as researchers become involved in action on the educative system. They must therefore regulate the actions that correspond to their initiatives according to a system of regulation prepared in advance.¹⁴

3.4 A form of research in didactics: « participatory observation »

Of course we may reject the theoretical model « research - action », while still accepting the education process as at once the subject of observation and the disposition of experience. No doubt it is not possible to completely separate:

- conceptions of interventions that aim directly at producing a decision or a procedure for accomplishing or improving teaching,
- conceptions that aim at the possibility of establishing a valid statement about teaching conditions.

Yet if we consider the tripartite « students - teacher - researcher » as a system, a minimum number of rules can be established. These will allow the system to work in terms of its obligations and regulation.

An example exists of such a system. Its construction calls for considerable effort, but its numerous results are both original and interesting. Here are a few rules governing this system of controlled, participatory observation.

The researcher conceives and chooses

- the new conditions in terms of the learning experience he would like brought about,
the exercises that allow verification of the intended acquisitions and of the minimum success to be obtained.

- a length of time for the intervention that includes time for the proposed lesson, and supplementary time for remedial actions should results prove insufficient.

It is best to compare teaching conditions (efforts, time necessary, etc) that give identical results.

The researcher proposes these conditions to a teacher (or a system of education). When he accepts, the teacher retains full authority to interrupt the experiment at any time, should he judge the contract is not being respected. He then takes over the work in his own way. Priority is given to insuring the best results for students, according to the criteria taken together. These results are followed up and verified within the establishment for the duration of a student’s schooling, together with the research organization and the teachers, whether or not there are experiments still in progress.

The researcher observes the group of processes and determines the realization of the conditions he proposed, as well as events that seem to result from it. Experimental phases on the whole effect rather short periods of time, if only because of the amount of work that goes into organizing the data. They represent at most 10 percent of teaching time.

In order to insure a certain stability of practices, the teachers (3 for 2 classes) do « lessons » prepared and written in common. They are able to present or continue either lesson in turn. Modifications are made in cooperation with the research organization. No a priori « common pedagogical doctrine » is postulated, none is rejected, but all realizations are discussed.

These dispositions preserve the functioning and regulation of teaching under the responsibility of the « education expert », i.e. the teacher. They are balanced by others that encourage cooperation with the researchers. For example, the teacher is recruited by the research organization with administrative agreement, according to a contract of limited duration, but renewable.
The cost and cumbersomeness of this system appears excessive today. Considering the diversity of ways opened to researchers, and the exploitation and diffusion of information gathered, this disposition hardly offers an encouraging perspective. However, it seems to me essential that in some instances the community should keep open the possibility of benefiting from this sort of relationship with mathematics education.

4. Relationship between the didactics of mathematics and mathematicians

80 percent of mathematics research consists in reorganizing, reformulating, and “problematizing” mathematics that have already been « done », by the researcher himself or by others. In the majority of cases the researcher also teaches mathematics, rarely the same ones, which again calls for reorganizing, reformulating, and “reproblematizing” the area of mathematics he intents to teach, in order to adapt the mathematics to his students and their needs. Of course, to create good mathematics, he pursues other activities and exhibits other qualities, distinct from those necessary to « teach well ». But to reorganize, reformulate and “reproblematize” an area of mathematics more or less vast, constitutes the essence of didactic activity as opposed to pedagogical activity, where the specificity of transmitted knowledge does not intervene. Thus all mathematicians are practitioners and consequently connoisseurs of didactics as applied to mathematics.

Inversely, all mathematics teachers are mathematicians, or at least in the broad sense suggested by Thurston: « I call mathematician anyone who develops (improves and extends) the human comprehension of mathematics »15.

4.1 Difficulties and failures of didactic activities

In spite of extensive development of the means of communication and of mathematics teaching, spontaneous didactic activity has its limits. Within the community of mathematicians is regularly heard the recrimination that good syntheses are too rare to
insure the verification of a consistent whole, threatened by the huge influx of disparate theorems. Without, others complain about the difficulties of diffusing mathematics in institutions where they are necessary, and about the « failure », on the part of a profession that should see to this need, to teach them. This holds true even in France, where the ratio of mathematics teachers is somewhat better than elsewhere. To know and practice didactics as an activity is no longer sufficient for mathematicians when it comes to answering both the needs of society and its appetite for control. In the same way the knowledge of economics put to use by store owners and housewives, hardly suffices when it comes to today’s economic questions. Moreover, just as there exist phenomena in economics, there exist didactic phenomena, irresistible yet unsuspected by those who act, phenomena whose correction demands analysis and comprehension. The domain that defines and takes charge of these phenomena is the didactics of mathematics, or the science of conditions specific to the diffusion of mathematical knowledge useful both to institutions and to men.

Mathematicians need a science in order to carry out their dual task of research, and education. They also need a science in order to assume their social responsibility, including an epistemological vigilance vis-à-vis the diffusion of mathematical knowledge. For when it comes to judging the quality of teaching in their discipline, society turns to them for the last word. Work accomplished in the domain of mathematics didactics is destined not only for teachers and researchers, but for the entire mathematical community. Part of its own efforts must be consecrated to these questions.

The ambition to understand and facilitate the didactic task of mathematicians and teachers calls for the use of diverse disciplines, as much for practical as epistemological reasons. This explains the diversity of researchers interested in the didactics of mathematics, and a certain confusion in its given representation. Yet as in economics, didactics has its unique object, to be approached in its own, independent way as compared to other approaches - psychological, sociological, linguistic, historical and so on. There is no reason to subordinate the didactics of mathematics in some non-exclusive way to any one of these disciplines, even if their usefulness in this domain is not contested.
Phenomena linked with didactics diffusion are not, by definition, independent of the knowledge diffused. Moreover, it is recognized both within and without the community that the diffusion of mathematics, and in particular the teaching of it, is the business of those who create, and not of those who repeat. In the study of processes through which mathematics is diffused, it is a question of describing or producing mathematics, and here mathematics is doubly implicated: at once as the subject of study, and as the means of these studies, although the subject and the means do not necessarily belong to the same area of knowledge. In these conditions, and a fortiori, the core of didactic of mathematics is necessarily a work for mathematicians, namely for some researcher they discuss with mathematicians inside of their community.

To link posts in the didactics of mathematics to mathematics itself is therefore advisable. I would add that it is in the interest of mathematicians to integrate the didactics corresponding to their specific area. For on the one hand many students of mathematics will not find employment in mathematics in the strict sense, ending up teachers of mathematics, and on the other because 80 percent of their activity with be none the less of a didactic nature.

To conclude, the decision on the part of important scientific universities to attribute, at a professorial level in mathematics, a post to the didactics of mathematics, seems to me an excellent and altogether judicious initiative.

The didactics of mathematics has developed in France mainly through the efforts of mathematicians provided with an IREM (Institut de Recherche pour l’Enseignement des Mathématiques)\textsuperscript{16}, in order to examine the teaching of mathematics, and to act upon this teaching in ways far more rewarding than through mere administrative decision. Scientific didactics, both theoretical and experimental, has progressively detached itself from immediate action on the practices and reforms of education, but profound affective and technical attachments subsist. These ties seem to me not merely a legacy but a necessity. In any case we set up our IREMs according to this conception.

For nearly 30 years now the didactics of mathematics has progressed considerably and stands ready for truly important advances. On the other hand diffusion has remained rather confined, whether vis-à-vis mathematicians, more interested in other
tasks, or vis-à-vis teachers and professors, caught up in storms of reform diverse in origin.

Other communities (specially in psychology or pedagogy) has a very false idea of our area of investigation.

5. Conclusion

I cannot conclude without reminding you that our work over the years has relied on an astounding number of people and researchers, both European and other. They have through very diverse means attempted to understand and improve mathematics education. I would have liked to name a few, but my choice could only be unfair. Today you might call them « precursors », but more often than not we haven’t as yet appreciated to the full extent their works, nor drawn the consequences of their successes and failures. The researchers of my generation sometimes feel they’ve left their young colleagues in a discouraging shambles of work, vast and very rich, but very disordered. Moreover, the perspectives for action are much narrower than those we found. I cannot encourage them enough, to continue examining a domain for which I have had an abiding passion, and which has given me so much joy.

Notes

1. reference to the Erlangen Program from Klein
2. reference to his conference of 1900 in Paris
3. or what we might have called didactology
4. An importance reflected in the name retained for a research institution we proposed at the ICME congress in Karlsruhe (together with Vergnaud and Fishbein, among others) viz., « Psychology of Mathematics Education ».
5. Too much so for P. Greco, a genetic psychologist who supported enthusiastically the first steps taken towards a didactics of mathematics (see articles in the Encyclopedia Universalis among others), only he did not believe in a scientific study!
6. Hypo-system: all relations relative to a few sub-systems
7. Infra-system: relations concerning the entirety of sub-systems

8. Because of these problems I found myself obliged to study the articulation between « didactic » phases, where the teaching intention is announced, and « adidactic » phases, where the teacher’s didactic intentions are provisionally unknown to the student, whether purposefully or not.

9. H. Steiner proposed the study of this problem by creating the TME group.

10. It is not proper here to take into consideration actions motivated by other reasons.

11. The study of the relationship between theory and practice has been undertaken by a group created by Bent Christiaensen.

12. We cannot help but evoke the work of J.M. de Ketele in this domain.

13. It engenders them, either by generalization - for example, an institution wishes to modify the mathematical means and procedures for decision making of another institution - , or by specifications or reductions of present elements - for example, the teacher has only one student, or else the teacher acts through the intermediary of diverse media, or else he becomes a fiction, as in autodidactism, etc.

14. Once again I call attention to the methodological error consisting in gross comparaisons of student success percentages and exercises proposed in the course of learning. These percentages are regulated variables of the system, they measure marginal differences.


16. and also the efforts of all those who pushed for the great reforms of the 60’s and 70’s in Europe: J. Dieudonné, G. Choquet, C. Gattegno, G. Papy, W. Servais, Anna Krigowska, Hans Freudenthal, preceded and followed by many others.

(Translated from french by William Fergusson)