SEMITRANSPARENT MIRRORS AS TOOLS FOR GEOMETRY TEACHING

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Abstract: *The author presents and discusses an experience in Geometry teaching, made in several years in co-operation with teachers of the Didactic Research Group of Trieste. This experience, based on the use of semitransparent mirrors, led to the realization of a Mathematics exhibition with didactic purposes, which is also described.*
Keywords: *-*

1. Laboratory activities and didactic tools

The Italian ministerial programmes for primary and middle school Mathematics (for 6-14 years old pupils) contain many geometrical topics, one particular topic being geometric transformations. As a first approach to this topic, the programmes suggest stimulating the pupils' curiosity and intuition, encouraging them to operate on and manipulate physical apparatus, the recommendation being to use a laboratory for Mathematics activities. Such a Geometry laboratory can be realized without necessarily using computers, by giving the pupils papers, scissors, ink, mirrors and everything necessary to make the first experimental observations about symmetries, folding and cutting papers and making ink spots. Following this suggestion, the teachers of the *Didactics Research Group of Trieste* (which is active in the Department of Mathematical Sciences of the Trieste University since 1970s and is financially supported by the Italian Ministry for University and Scientific and Technological Research and by the Italian National Council for Research) use, in laboratory activities, both electronic instruments (from the simple pocket calculator to the personal computer) with different kinds of software (mainly Cabri for Geometry teaching), and didactic tools made using structured or commonly used materials. In fact the concept of didactic tool is very general: any object may be a didactic tool, if it is used in

appropriate way and if it is integrated in a well-planned teaching path. This "definition" of tool is restricted only to material objects, but we could also consider some learning environment as a "didactic tool" in an extended meaning, when this one really improves the learning: this is the case of the Geometry exhibition illustrated in the following, in which many factors (among them, the emotional factor to be exceptionally "out of the school") stimulate and motivate the pupils to learn.

2. A didactic tool for geometry teaching: the Simmetroscopio

The primary school teacher Bruno Giorgolo, member of the *Didactics Research Group of Trieste* (DRG), often realizes interesting didactic tools, uses them in his classroom activities and proposes them to the Group. Some of these tools are even commercially marketed. The main characteristic of the tools realized by Giorgolo and of the activities which are related with them is that they stress both the affective and playful aspects and the operative ones. One very interesting didactic tool created by Giorgolo around 1987 and continuously developed by him is called the *Simmetroscopio* (see: Giorgolo B. 1992); its first production and marketing in Italy was made by M.C.E. ("Movement of Educational Co-operation"). The *Simmetroscopio* has been used by the DRG since the school year 1987/88. It was presented for the first time in a Italian meeting in Torino in 1989 and since that was illustrated several times in many parts of Italy and to researchers of other countries. The *Simmetroscopio* is a simple and flexible instrument, easily adaptable to didactic itineraries, formed by a set of two or more semitransparent mirrors (made by treated glass) that can be assembled, rotated and shifted in various ways on a support. The support is also a work-desk, with other useful accessories. By such means we can treat a range of geometrical concepts and develop various didactic itineraries about Geometry, to study the fundamental objects and properties of Euclidean Geometry, and various geometrical transformations of the plane or of the space. Of course, it is also possible make significant links with other disciplines, such as Physics, and use this instrument in other activities, for instance the expressive ones. There exist many types of Simmetroscopio, studied for different aged pupils. It is used at any level, from 5 years old pupils to adults. In fact, many teachers of our Group tested it at primary and middle school, we have used it also in courses for teachers and Giorgolo has also adapted it to kindergarten level (see: Giorgolo B. 1997); in this last

form it was used in experiments in some municipal kindergartens of Modena (Italy). Below some pictures of the Simmetroscopio are shown; the semitransparent mirror permits not only the reflected image to be seen but also the objects which lie beyond itself. This is useful for many didactic purposes. To stress the semitransparency, in figures 1 and 2 symmetric objects of different colours are posed on opposite sides of the mirrors and we see in each mirror an object coloured by composition. There exist other tools that use the semitransparent effect; see for instance: Lott J.W. 1977.

Fig.1: The basic version of the Simmetroscopio

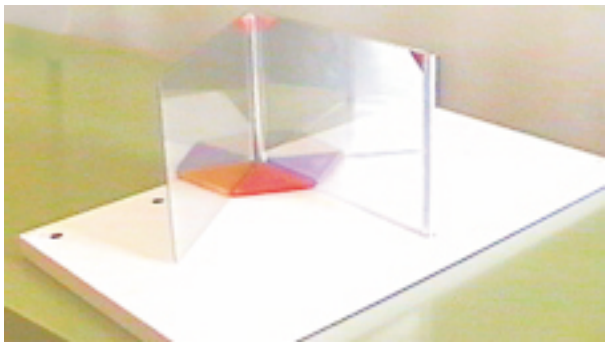

Fig. 2: A version for learning rotations

3. The exhibition "Oltre Lo Specchio" ("Beyond the mirror")

The experiences of the Group using the Simmetroscopio led Giorgolo and me to the planning of an exhibition structured as a laboratory, based on its use (see: Zuccheri L. 1992, II), that is the exhibition "*Oltre lo specchio*" ("*Beyond the mirror*"). It was realized in 1992 in co-operation with the *Laboratory of Scientific Imaginary of Trieste* and from 1992 until 1997 it was visited by many thousands of people. It is a *hands-on* exhibition (the British psychologist Richard Gregory coined this word), that means it is a learning environment, placed outside the school, in which everything shown must incite interest and curiosity and, in contrast to what happens in most traditional museums or exhibitions, everything must be touched (for references about this type of museums and exhibitions, see: Salmi H.S.1993).

"*Beyond the mirror*" was developed as a teaching aid for primary and middle school teachers about geometrical transformations, therefore its contents are the ones contained in the primary and middle school programmes and it is structured by pre-established learning paths. It deals with various geometrical subjects, e.g. reflections, rotations, similarities, projective transformations, and each of them, for a methodological belief tied to the exhibition context and linked to practical problems (such as length of visits, and pupils' attention span), is treated in various deepening levels. The methodology used is that of the guided discovery through the opportune development of activities (see: Zuccheri L. 1996): the exhibition is in fact a Mathematics laboratory in which visitors find paths of various levels of difficulty that conduce them to discover and learn particular Mathematical concepts, by means of observation, object manipulation and drawing. The activities are progressive and they go on from a very concrete operative level to a more abstract one. They are individual, two by two, or group ones. The two by two activities often request that the two pupils interact and that they exchange each other questions, observations and reflections. The various activities are grouped in sections, each of them has a structure conceived to motivate the pupils to learning. Each section is composed of various work stations, with different activities; here there are the instructions cards, explaining the work to do, and all the necessary materials (semitransparent mirrors, structured materials, various objects of common use, operative cards with printed drawing, writing-materials). Usually, a very simple task to do with an amusing individual or group activity (the play element is an essential component of the methodology) is proposed as the first one; very often, during this first activity the pupils make observations about their body, and about phenomena that will be analysed later, re-proposed in concrete objects or drawing. A poster summarizes the essential points, which the section work aimed at, with the aim to fix the fundamental concepts. To achieve the most effective results, various communication levels are used: representative iconic, verbal written, verbal spoken. The last one is the task of the appropriately prepared assistants, who guide exhibition visitors, giving them the necessary directions and suggestions. Some of the main contents and some paths developed in the exhibition are described as follows (see: Zuccheri 1992, I).

- *Reflection symmetries path*: this starts with practical experiences with giant mirrors in three-dimensional space, and then in the plane, observing also the

clockwise/anticlockwise rotation change; the concepts are later formalized in two deepening levels (one for primary and one for middle school); the most difficult level conduces the visitors to solve geometrical problems.

- *Rotations and angles path*: this starts with practical experiences in three-dimensional space with a giant kaleidoscope, and then in the plane; the concept of angle measure and the composition of two axial symmetry with intersecting axis are also treated.
- *Translations path*: this is carried out starting from three-dimensional space with the observation of the effect of two big parallel mirrors, and later considering the plane; the formalization of the concepts is given then in two different levels; the most difficult one leads to the proof of the fact that the composition of two symmetries with parallel axes is a translation.
- *Similarities transformations path*: similarity is treated at the operative level for younger pupils, giving them the instructions to reduce or magnify some figures; a deeper level conduces the pupils to understand the underlying geometrical concepts.

4. Some examples

In this section we describe some exercises among those presented to the Thematic Group "Tools and Technologies in Mathematical Didactics" and during the Poster Session. They belong to the set of exercises from the exhibition "*Beyond the mirror*" (see: Zuccheri L. 1995). Only the kernel concepts of each exercise are sketched.

4.1 An exercise that mathematicians like

Given drawings like in figure 3 the pupils have to complete them with respect to their symmetry axis, on both the sides, using the semitransparent mirror. The piece of paper must be arranged below the mirror, overlapping the axis of symmetry with the bottom border of the mirror, and cannot be moved. Then the pupils operate on both sides of the mirror, copying the images that they see in the mirror. From psychomotorial point of view, we observe that it is easy to copy a drawing on the side beyond the mirror,

whereas it is difficult to copy a drawing on our side of the mirror. To make this task easier, we could permit the pupils to rotate the paper under the mirror and to draw always beyond it. The mathematicians like this exercise, especially in the version described before, because this type of activity stresses the mathematical model of symmetry as transformation of the whole plane (i.e. one-to-one correspondence of the plane, as set of points, into itself); other implementations of this model (i.e. paper folding, normal mirrors) don't work so well, also from psychological and perceptive point of view.

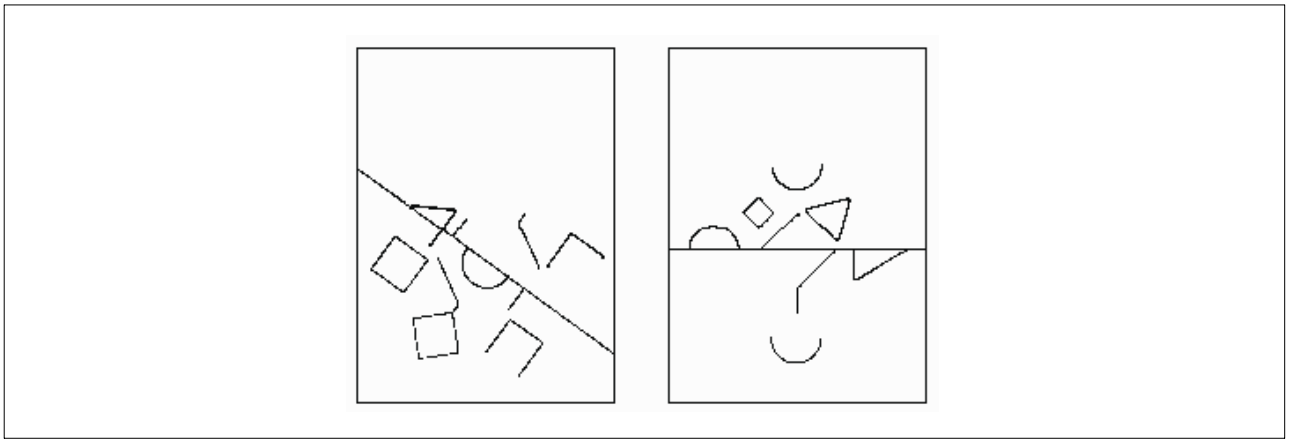

Fig. 3: Some operative cards from the exhibition "*Beyond the mirror*"

4.2 A path for describing the composition of two planar axial symmetries with parallel axes

The pupil proceeds starting from step 1 and he or she stops when the selected difficulty level adapted is reached. The difficulty level is linked to the age and to the previous knowledge of the pupil. Usually steps from 1 to 4 are for primary school pupils (steps 3-4 only for the last year), whereas the pupils of middle school can continue up to the end.

- *Step 1: Observation in three-dimensional space.* The pupil observes the effect of reflection of some object or of the body of a classmate in two giant parallel mirrors.

- *Step 2: Observation in the plane.* The pupil observes the effect of the reflection of a drawing representing a car (like figure 4.a), which is posed on the table, between two parallel mirror; it seems to be riding toward one of them; the pupil looks in this mirror and observes the various different images which are formed by the reflections. The first image seems to be a car which is going toward the real car figure, whereas the second image seems to ride in the same direction of the real car figure. The bottom borders of the two mirrors are depicted in different colours (red and green) and this is useful to represent the symmetry axis in the plane of the table.
- *Step 3: Concrete experience to link the empirical notion of translation as motion with that of translation as geometrical transformation of the plane.* The pupil repeats the previous observations using semitransparent mirrors and a cardboard model which represents a short motor racing track (see figure 4.b). A transparent figure of a car can be moved along the track, starting from the initial position, in which it overlaps an identical car figure. First the pupil puts the cardboard model on the table, between the mirrors (and the two cars overlap each other), then he moves the transparent car passing under one of the mirrors and looking in it. When the moving car passes over the first reflected image, he observes that the car doesn't overlap it (the reflection is an opposite congruence). When the moving car passes over the second image, the pupil observes that the car perfectly overlaps it (the reflection of the first reflected image is the translated image of the car in the initial position, and the translation is a direct congruence). Finally the pupil measures by a rule the distance between the two mirrors, and the distance between the starting and ending lines of the track, which are drawn in the model. He or she compares them and observes that the track is twice as long as the distance between the mirrors.
- *Step 4: First generalization*. To avoid the conceptual error of confusing the translation direction with some preferential direction suggested by the figure itself (the simplified previous model uses a car which is directed in the translation direction) the pupil considers a drawing like figure 4.c, puts it on the table posing the car figure between the two semitransparent mirrors as before, and copies the first two reflected images which he sees in one of the mirrors. Then he or she measures by the rule the distances between various points of the printed drawing and the correspondent ones on the second drawing that he has made. The pupil

observes that the distance is always the same (and that it is the same as found before). The idea of this strange movement is given to the pupil saying that the car is moved by a strong gust of "Bora" (the strong wind of Trieste).

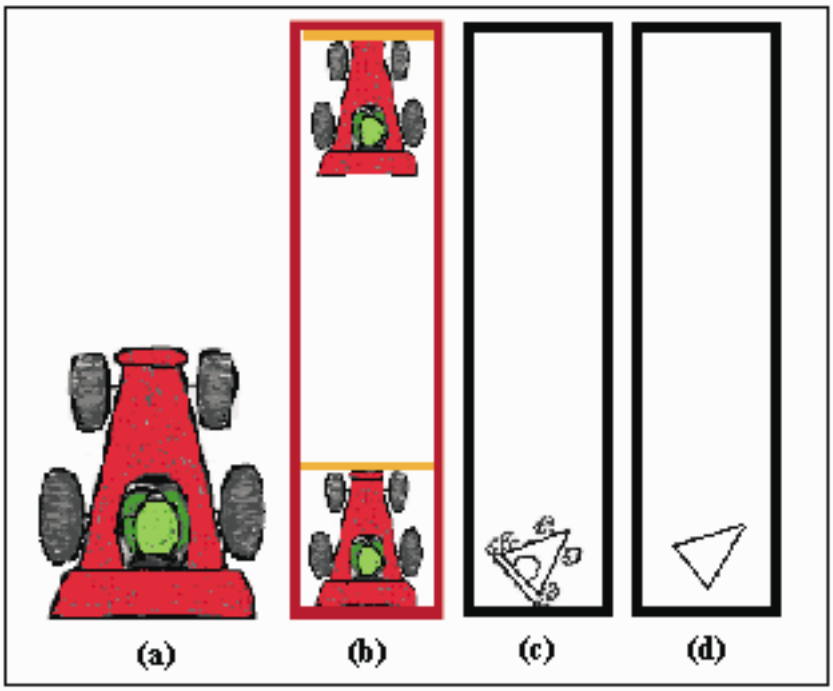

Fig. 4: Various steps of the translation paths

- *Step 5: Formalization*. The figure 4.c becomes more schematic, like figure 4.d. The previous exercise is repeated with figure 4.d, using semitransparent mirrors and drawing the two first triangles reflected into one of the mirrors. The vertices are labelled writing A, B, C on the printed figure, A',B',C' on the correspondent points of the first reflected image, A", B", C" on the correspondent points of the second image. The symmetry axes are depicted copying the bottom border of the mirror (red, we suppose) and the first green line which appears beyond the mirror.
- *Step 6: Problem posing*. The pupil observes that, if the paper sheet (on which he or she has just drawn) is moved in a direction perpendicular to the mirrors, then the first image does not overlap the triangle A'B'C', whereas the second figure always overlaps the triangle A"B"C". We ask the pupil why this fact happens.
- *Step 7: The solution*. The guided discovery path ends with the explanation of the phenomenon just observed (from mathematical point of view). The pupil reads on

the instruction card the correct explanation, i.e. the proof of the fact that the composition of two axial symmetry with parallel axes is a translation, with given direction and length. The pupil can compare his or her own reasoning with this explanation. The explanation is given using the formal symbols we have introduced before and geometrical arguments which involve the general properties of axial symmetries. This is a real mathematical proof because the reasoning is valid for any point of the plane.

5. Conclusion

In the experiences carried out by the Didactics Research Group of Trieste using the Simmetroscopio, it seems clear that the semitransparent mirrors offer a pleasant and immediate approach, both to children and to adults. Such mirrors give the pleasure of discovery, make the intuition of a range of geometrical concepts easier and, especially, can be use by students in an autonomous way, since they allow self-correction, with a remarkable feedback effect. The didactic power of the semitransparent mirror of Simmetroscopio is that it provides the possibility of seeing a reflected image and, at the same time, acting over this image, copying it beyond the mirror, or modifying it, drawing on our side of the mirror. This tool, integrated with the didactic paths of the Geometry laboratory "*Beyond the mirror*", revealed itself very effective for many purposes, and not only for the basic ones for which the exhibition was carried out. In fact the exhibition was also visited by classrooms of higher school level, was used fruitfully to up-date teachers of primary and middle school, and to prepare future teachers in a University Didactics course. The courses for teachers allowed the teachers to continue, in their classrooms, the development and the deepening of the geometrical concepts illustrated in the exhibition and various written materials were prepared for this purpose (see references below). The exhibition assistants often made reports about their work in the exhibition and this was fruitful to improve some exercises. The most meaningful practical result achieved was that many teachers integrated their Mathematics curricula with various visits of the same class, in different times, to the exhibition itself. The exhibition was appreciated from experts of this field; it was mentioned in the report about Italian Research in Mathematics Education presented at ICME 8 (Seville, 1996) (see: Fiori C, Pellegrino C. 1996; M.Marchi, A.Morelli &

R.Tortora 1996).

Acknowledgement

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